FORMALIZING AN ABSTRACT ALGEBRA TEXTBOOK IN ISABELLE/HOL

J. ARANSAY AND J. DIVASÓN

Abstract. In this work we present the formalization of some well-known results in Abstract Algebra with the proof assistant Isabelle/HOL. Our interest focuses in finite-dimensional vector spaces, and our proposal is to closely follow the presentation made in a popular book of the field by P. R. Halmos. The main result in this work proves that every finite-dimensional vector space $V$ over a field $\mathbb{K}$ is isomorphic to $\mathbb{K}^n$, with $n$ the dimension of $V$. In the way to this result we had to make relevant choices on the representation of algebraic structures in the theorem prover, from which we inform in the paper. We also report on some informal "pen & pencil" proofs and arguments presented in Halmos, that we had to turn into algorithms for the proof assistant to accept them. The work fills a gap in our project of computing properties of generic finite-dimensional vector spaces by means of matricial algebra over fields, since it formally proves that both constructions are isomorphic.

Departamento de Matemáticas y Computación, Universidad de La Rioja; Edificio Luis Vives, c. Luis de Ulloa – 26004. La Rioja. Spain.

E-mail address: {jesus-maria.aransay}, {jose.divason}@unirioja.es

This work has been supported by project MTM2009-13842-C02-01 (Ministerio de Educación y Ciencia) and by FORMATH project, nr. 243847, of the FET program within the FP7 of the European Commission.