Workshop on Geometric and Topological Combinatorics

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Abstracts of the poster session

List of posters

SMALL CHVATAL RANK OF AN INTEGER MATRIX

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Abstract

Given an integer matrix B, the Chvatal rank of B is the minimum number k such that for every right-hand side b, the integer hull of the rational polyhedron $\{x : Bx \leq b\}$ can be obtained with at most k rounds of cutting planes. Cutting plane methods were introduced by Gomory and further developed by Chvatal, Schrijver and others.

Motivated by this definition, we define the notion of "small Chvatal rank" (SCR). For each full-rank minor B_σ of B, form the Hilbert basis of the cone spanned by B_σ. Let $B^{(1)}$ be the union of these Hilbert bases over all such minors. Do the same with $B^{(1)}$ replacing B to form $B^{(2)}$, and so on. The SCR of B is the minimum number k such that $B^{(k)}$ contains all facet normals of integer hulls of polytopes of the form $\{x : Bx \leq b\}$ as b varies.

The SCR is easily bounded above by the Chvatal rank, but sometimes it can be much smaller. For example, if B is a $2 \times n$ matrix then the SCR is always at most 1, while the Chvatal rank can be arbitrarily large. We will explore and contrast the two ranks for several families of examples. SCR has applications to secondary fans and Groebner fans of point configurations.

ON IRREDUCIBLE TRIANGULATIONS OF 2-PSEUDOMANIFOLDS

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Abstract

It is known that every triangulation of minimum degree at least four of a compact surface without boundary can be obtained from an irreducible triangulation by an easy method that combine a sequence of vertex splitting and addition of octahedron. The set of irreducible triangulations (under these operations) of the sphere, the torus, the projective plane and the Klein bottle are also found in the literature ([1], [2], [3], [4], [5]).

We extend these results to the case of compact 2-pseudomanifold with (possibly empty) boundary. In particular, we give irreducible triangulations of the cylinder, the Möbius strip, the punctured torus, the punctured Klein bottle, the strangled torus and the strangled Klein bottle.

Keywords: Triangulation; compact 2-pseudomanifold with boundary; vertex splitting; contraction of edges.

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SUBCLASSES OF INTERSECTION GRAPHS OF PSEUDOSEGMENTS

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Abstract

A graph is a PSI-graph if there is a one-to-one correspondance between the set of vertices and a set of pseudosegments such that two vertices are adjacent if and only if the corresponding pseudosegments intersect. The recognition problem of the class of PSI-graphs is NP-complete. We analyse which graph properties do imply or prevent such a representation.

Interval graphs are defined similarly and can easily be transformed into PSI-representations. A superclass of interval graphs are chordal graphs. We show that the subclass of VPT-graphs known as path graphs belongs to the class of PSI-graphs and that there is a family of chordal graphs that does not admit a PSI-representation.

Another superclass of interval graphs are cocomparability graphs. We can construct PSI-representations for some special cases of cocomparability graphs of orders of interval dimension 2.

HOM COMPLEXES AND HOMOTOPY THEORY IN THE CATEGORY OF GRAPHS

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Abstract

We investigate a notion of ×-homotopy of graph maps that is based on the internal hom associated to the categorical product in the category of graphs. We show that graph ×-homotopy is characterized by the topological properties of the so-called Hom complex, a functorial way to assign a poset to a pair of graphs. Along the way, we also prove some results that describe the interaction of the Hom complex with certain graph theoretical operations, including exponentials and arbitrary products. Graph ×-homotopy naturally leads us to a notion of weak equivalence which we show has several equivalent characterizations. We describe the relationship between weak equivalence and the graph operation known as (un)folding, and apply the notions of weak equivalence to a class of graphs we call contractible (dismantlable in the literature) to get a list of conditions that again characterize these. We end with a discussion of graph homotopy arising from the internal hom associated with the Cartesian product; in the category of reflexive graphs this construction is known as A-theory in the literature.

THE STRUCTURE OF FLAG SPHERE TRIANGULATIONS

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Abstract

The combinatorial structure of a simplicial complex is (partially) described by its f-vector, i.e. the n-tuple $(f_0, f_1, \ldots, f_n)$, where f_i denotes the number of i-dimensional simplices. The goal is to characterize those n-tuples which come as f-vectors of flag triangulations of the 3-dimensional sphere (a simplicial complex is called *flag* if together with any 1-skeleton of a simplex it contains the simplex itself).

[1] conjectures that a flag 3-sphere triangulation either has the f-vector of a complex obtained from an iterated edge subdivision of the hiperoctahedron or is the combinatorial join of an n-gon with an m-gon, where n and m are close. I give an elementary, geometric proof of this conjecture, up to (as of 29.06.2006) finite (and less than 10) number of cases.

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COMPUTING THE NEWTON POLYTOPE OF THE RESULTANT

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Abstract

The Newton polytope $N(\mathcal{R})$ of the resultant of a system of polynomial equations with corresponding Newton polytopes $P_0, \ldots, P_n$, is the convex hull of the exponent vectors of its monomials. Sturmfels established an one to one and onto correspondence between the extreme monomials and the mixed cell configurations of the Minkowski sum $P = P_0 + \cdots + P_n$. By means of the Cayley trick the problem of enumerating all regular fine mixed subdivisions of P is reduced to enumerating all regular triangulations. The latter are in bijection with the vertices of the secondary polytope. The algorithm proposed by Imai et al., uses reverse search techniques to compute a spanning tree of the secondary polytope. We propose a modification of this algorithm to enumerate all mixed cell configurations and thus to compute the extreme monomials of the resultant. An application to implicitization is given and the possibility of enumerating only the vertices of the resultant polytope $N(\mathcal{R})$, which lie on its silhouette with respect to a canonical projection π, is explored.

Θ-CRITICAL SETS OF LATIN SQUARES HAVING Θ AS A PRINCIPAL AUTOTOPISM

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Abstract

Any principal autotopism Θ of a Latin square $L \in LS(n)$, whose elements are in a set N of n symbols, gives a significant information about the symmetry of L. Although Θ-critical sets of L can be then used in Cryptography to get the access structure of a secret sharing scheme [1, 2, 4], the size of the smallest one is still an open problem. Because Θ can be decomposed into triples of a partial Latin square [3], we propose in this paper an algorithm depending on the order of L allowing to give an upper bound of the previous size. This algorithm reduces the previous problem to the calculus of the size of the smallest critical set of a Latin subrectangle of L of order $k \times n$, which can be decomposed at the same time into k regions, each of them having all the symbols of N. Some general results using non principal autotopisms are also given.

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AXIOMATIZATION OF CHIROTOPES OF PLANAR FAMILIES OF PAIRWISE DISJOINT BODIES

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Abstract

We show that a map defined on the set of triples of a finite set of indices is the chirotope of a planar family of pairwise disjoint bodies if and only if its restrictions to subsets of triples of subsets of indices of size at most five are chirotopes of planar families of pairwise disjoint bodies. The proof goes through the introduction of the class of *arrangements of double pseudolines*, through a generalisation of an homotopy theorem for arrangements of pseudolines (G. Ringel, 1953) to arrangements of double pseudolines, through a finite characterisation of the isomorphism classes of arrangements of double pseudolines, and at last through putting in one to one correspondance the isomorphism classes of arrangements of double pseudolines and the chirotopes of planar families of pairwise disjoint bodies.

ON THE TOPOLOGY OF THE FREE COMPLEX OF A CONVEX GEOMETRY

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Abstract

In a convex geometry $\mathcal{C}$ on a finite set X, its free sets form a simplicial complex $\mathrm{Free}(\mathcal{C})$. This complex is known to be nonevasive, thus contractible. Further, the topology of $\mathrm{Del}_{\mathrm{Free}(\mathcal{C})}(v)$ is investigated by Edelman and Reiner (2000) for some special cases. We investigate this complex for general convex geometries and show the following, answering the conjecture and the question asked in their paper.

(1) If $\mathrm{Dep}(v) \neq X$, the deletion $\mathrm{Del}_{\mathrm{Free}(\mathcal{C})}(v)$ is nonevasive.

(2) If $\mathrm{Dep}(v) = X$, $\mathrm{Del}_{\mathrm{Free}(\mathcal{C})}(v)$ can be homeomorphic to any finite simplicial complex.

FINDING MINIMUM WEIGHT TRIANGULATIONS

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Abstract

The minimum-weight triangulation problem is the problem of finding a triangulation, for an input planar point set, which minimizes the sum of Euclidean lengths on the edges present. A very recent breakthrough, by Mulzer and Rote, asserts that the problem is actually NP-hard. Due to this, it is of interest to find practical ways of finding a minimal triangulation in given instances. A novel approach via graph preprocessing and integer and linear programming is presented along with a report on experimental results.

GRAPHS OF TRANSPORTATION POLYTOPES

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Abstract

Transportation polytopes are well-known objects in operations reseach and mathematical programming. These polytopes have very quick tests for feasiblity, coordinates of a vertex can be quickly determined, and they have nice embedding properties [2]. Using the notion of chamber complex, Gale diagrams, and the theory of secondary polytopes [1] we are able to exhaustively and systematically enumerate all combinatorial types of nondegenerate transportation polytopes of small sizes. These generic polytopes (those of maximal dimension whose vertices are simple) will have the largest graph diameters and vertex counts in their class. Using our exhaustive lists, we study some of the conjectures of Yemelichev, Kovalev, and Kratsov [3]. In particular, our study is centered on questions related to the 1-skeleton graph of these polyhedra.

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EVERY POINT IS CRITICAL

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Abstract

For any compact Alexandrov surface S with curvature bounded below and without boundary, and any point y in S, let Q_y^{-1} denote the set of all points x in S, for which y is a critical point; i.e., for any direction v of S at y there exists a shortest path from y to x whose direction at y makes an angle $\alpha \leq \pi/2$ with v.

We prove $\mathrm{card}Q_y^{-1} \geq 1$, and show that equality for all $y \in S$ characterizes the surfaces homeomorphic to the sphere. For orientable surfaces we show that Q_y^{-1} is finite, and satisfies $\mathrm{card}Q_y^{-1} \leq 5$ if the genus g of S is equal to 1, and $\mathrm{card}Q_y^{-1} \leq 8g - 4$ if $g \geq 2$. Our last result shows, roughly speaking, that for orientable surfaces the sets Q_y^{-1} have generically an odd number of elements. Some open questions are presented.

Since every farthest point from x is critical with respect to x, our results also contribute to a description of farthest points H. Steinhaus had asked for.

THRESHOLD BEHAVIOR OF THE HOMOLOGY OF THE CLIQUE COMPLEX

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Abstract

In a seminal paper, Erdős and Renyi exhibited a sharp threshold function for the connectivity of the random graph $G(n,p)$. They showed that if $p \gg \log(n)/n$ then $G(n,p)$ is almost always connected, and if $p \ll \log(n)/n$ then $G(n,p)$ is almost always disconnected as n approaches infinity.

The clique complex $X(G)$ of a graph G is the simplicial complex with all complete subgraphs of G as its faces. In contrast to the zero dimensional homology of this complex, which measures the connectivity of the underlying graph, the higher dimensional homology groups do not correspond to monotone graph properties. We show that there are nevertheless higher dimensional analogues of the Erdős-Renyi Theorem for $X(G(n,p))$.

For each k we find constants $-2 < c_1 < c_2 < c_3 < 0$, depending on k, satisfying the following. If $p = n^{\alpha}$, with $\alpha < c_1$ or $c_3 < \alpha$, then almost always the k-dimensional homology of $X(G(n,p))$ is trivial, and if $c_1 < \alpha < c_2$, then almost always the k-dimensional homology is nontrivial. We also give estimates for the expected rank of homology.

CHARACTERIZATION OF THE PACKING PROPERTY OF THE CLUTTER OBTAINED FROM A POINT CONFIGURATION

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Abstract

We investigate the clutter of the positive cocircuits of the oriented matroid obtained from a point configuration. We show that the clutter of positive cocircuits obtained from a point configuration whose convex hull is a cross polytope has the packing property if and only if it does not have the clutter of minimal vertex covers of an odd cycle as a minor. Moreover, we show that the clutter of positive cocircuits obtained from a point configuration whose convex hull is a simplex has the packing property if and only if it does not have an odd circular clutter C^2_{2k+1} as a minor.

EUCLIDEAN SIMPLICES AND GEOMETRIC INVARIANTS OF 3-DIMENSIONAL MANIFOLDS

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Abstract

It is well-known that any two triangulations of a piecewise-linear 3-manifold can be transformed into each other using local Pachner moves. If we construct an algebraic expression depending on some values ascribed to a manifold triangulation and invariant under these moves, we get a value that does not depend on a specific triangulation. It is natural way to construct a PL-manifold invariant. For example, Euler characteristic and Turaev-Viro type invariants can be proved to be invariant using exactly this method.

We present a new 3-manifold invariant. Its construction is naturally divided into three main parts. First, on a given representation of the fundamental group we build a covering of a 3-manifold corresponding to the kernel of the representation. Then, we make "geometrization" of the covering, i.e., map it into 3-dimensional Euclidean space. In the last, algebraic part, we build an acyclic complex. The invariant of all Pachner moves is expressed in terms of the torsion of that complex.

One can consider the simplest version of this invariant corresponding to the trivial covering of a manifold. Calculations show that this version raised to the $(-1/6)$th power equals the order of the torsion subgroup of the first homology group. However, when we use the maximal Abelian covering, which corresponds to the commutator of the fundamental group, we get more interesting version of the invariant associated with the (Abelian) Reidemeister torsion.

TOUCHING HOMOTHETIC BODIES – ON A CONJECTURE OF K. BEZDEK AND J. PACH

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Abstract

The following conjecture of Károly Bezdek and János Pach is cited in [1]. If $K \subset \Re^d$ is a convex body, then any packing of pairwise touching positive homothets of K consists of at most 2^d copies of K. We prove a weaker bound, 2^{d+1}. We also show two other results. One is that the bound 2^d holds in the case K is a cube. The other is that the stronger condition that all copies of K share one point in common on their boundary also implies the upper bound 2^d.

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NEW LOWER BOUNDS FOR THE NUMBER OF ($\leq k$)-EDGES AND THE RECTILINEAR CROSSING NUMBER OF K_n

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Abstract

We provide a new lower bound on the number of ($\leq k$)-edges on a set of n points in the plane in general position. We show that the number of ($\leq k$)-edges is at least

$$3\binom{k+2}{2} + 3\binom{k - \lfloor \frac{n}{3} \rfloor + 2}{2} \qquad \text{for } k \geq \lfloor \tfrac{n}{3} \rfloor.$$

As a main consequence, using results from [4], we obtain a new lower bound on the rectilinear crossing number of the complete graph or, in other words, on the minimum number of convex quadrilaterals determined by n points in the plane in general position. We show that the crossing number is at least $\frac{41}{108}\binom{n}{4} + O(n^3)$. Since $\frac{41}{108} = 0.37\overline{962}$, this improves the previous bound of $0.37533\binom{n}{4} + O(n^3)$ in [3] and approaches the best known upper bound $0.380739\binom{n}{4}$ in [2]. The proof is based on a result about the structure of sets attaining the rectilinear crossing number, for which we show that the convex hull is always a triangle. The techniques developed allow us to show a similar result for the halving-edge problem: For any n there exists a set of n points with triangular convex hull that maximizes the number of halving edges.

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EMBEDDING 3-POLYTOPES ON A SMALL GRID

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Abstract

We improve the upper bound for realizing 3-polytopes with integer coordinates. The 2d embedding of the 1-skeleton of the polytope can be found with Tutte's method. A careful analysis of the embedding leads to the location of the boundary points and the size of the stress on the boundary edges. The analysis uses the fact that the graph of the polytope can be substituted by a K_4 or K_5 with non-resolving forces. We use the Matrix-Tree Theorem and new bounds for the number of spanning trees of planar graphs to scale the embedding to integer stress and integral 2d coordinates. Finally we apply the Maxwell-Cremona correspondence to lift the stressed graph into 3d.

With our method the upper bound for the grid size is reduced to $O(188^n)$. The best known bound before was $O(18^{n^2})$ due to Richter-Gebert.

DIRICHLET STEREOHEDRA FOR CUBIC GROUPS

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Abstract

What is the maximum number of facets of a convex polytope that tiles $\mathbb{R}^3$ by congruent copies? [3, 4, 1]. Specially interesting is the *stereohedral* case, in which a group of isometries of $\mathbb{R}^3$ (necessarily a *crystallographic group*) acts transitively over the tiles. A general theorem (Delone, 1961) says that the number of facets of a 3-dimensional stereohedron cannot exceed 390 but none is known with more than 38 facets (Engel, 1980).

Engel's stereohedron is a *Dirichlet stereohedron*, that is, the tiling is the Voronoi diagram of an orbit of points under a crystallographic group. In this work we continue the intensive study of Dirichlet stereohedra started by the second author and D. Bochiş, who showed that Dirichlet stereohedra cannot have more than 80 facets except if their tiling group is cubic.

Taking advantage of the recent, simpler, classification of cubic groups [2] into 27 "full" (14 without reflections) and 8 "quarter" groups we prove:

Theorem. For full groups, Dirichlet stereohedra have at most 25 facets (and there is one with 17). For quarter groups they have at most 105 facets.

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POLYHEDRAL APPROACH TO PARTITIONS OF NUMBERS

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Abstract

We describe new, polyhedral, approach to the classical problem of partitions of numbers proposed by the author [1]. We give full description of the facets of the partition polytope P_n. Studying its vertices allows to reduce the huge amount of partitions of n to the vertices of P_n, which should help to solve optimization problems on partitions.

The polytope P_n is the convex hull of the set of the incidence vectors of all partitions $n = x_1 + 2x_2 + \ldots + nx_n$. The non-trivial facets are basic feasible solutions of rank $n - 2$ of a system of subadditive inequalities and equalities. Sufficient and, separately, necessary conditions for a partition to be a vertex of P_n are proved. Being executed to a computer program, together with quick by hand examination, they enable us to list all vertices of P_n for up to $n = 25$. We also show how the vertices and facets of the polytopes of constrained partitions — in which some numbers are forbidden to participate — can be obtained from those of P_n.

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A CHARACTERIZATION OF THE FACES OF THE GENERALIZED CLUSTER COMPLEX $\Delta^m(\Phi)$

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Abstract

Let Φ be a finite irreducible root system of rank n with associated reflection group W. Let Φ^+ be a positive system for Φ with corresponding simple system Π. The *generalized cluster complex* $\Delta^m(\Phi)$ is a simplicial complex on the vertex set of colored almost positive roots $\Phi^m_{\geq -1}$, that is the set consisting of m (colored) copies of each positive root and one copy of each negative simple root.

Consider the partition $\Pi = \Pi_+ \cup \Pi_-$ of the set of simple roots into two disjoint sets such that the roots within each one are pairwise orthogonal. We let γ be a bipartite Coxeter element with respect to the above partition and we consider a certain total order of the roots in Φ depending on γ. If σ is a face of $\Delta^m(\Phi)$ we denote by σ^i the subset of σ consisting of positive roots of color i and we let $\sigma_\pm = \sigma \cap (-\Pi_\pm)$. If $\tau \subseteq \Phi^m_{\geq -1}$ such that either $\tau \subseteq (-\Pi)$ or τ consists of positive roots of the same color, we denote by w_τ the product of reflections throught the roots in τ in decreasing order.

If T is the set of all reflection in W we denote by $l_T(w)$ the smallest k such that w can be written as a product of k reflections in T. We partially order the elements of W by letting $u \leq v$ if and only if $l_T(u) = l_T(u^{-1}v) + l_T(v)$. The faces of $\Delta^m(\Phi)$ can be characterized by the following criterion.

Theorem: The set $\sigma \subseteq \Phi^m_{\geq -1}$ is a face of $\Delta^m(\Phi)$ if and only if $w = w_{\sigma_+} w_{\sigma^1} w_{\sigma^2} \cdots w_{\sigma^m} w_{\sigma_-} \leq \gamma$ and $l_T(w) = |\sigma|$.

Using the above criterion, we deduce that $\Delta^m(\Phi)$ is shellable and $(m+1)$-Cohen-Macaulay.

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VORONOI CIRCLE OF THREE ELLIPSES

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Abstract

Computation of the Voronoi diagram of a set of ellipses in the plane involves high degree polynomial equations. We have solved efficiently several predicates, but the main operation appears to be the InCircle predicate, that is to determine the relative position of a query ellipse with respect to the Voronoi circle of three given ellipses.

We study different formulations of the underlying algebraic equations both in cartesian and parametric space. These formulations have lead us to a tight upper bound on the number of complex tritangent circles to three ellipses. We show the difficulty of solving the underlying algebraic system. In a previous work we have proposed a subdivision method to solve this system.

Here, we focus in pure algebraic methods. We present a way to quickly compute the resultant of the algebraic system that leads to an efficient implementation handling both degenerate and non-degenerate configurations.

VISUALISING f- AND FLAG VECTORS OF POLYTOPES

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(partially joint work with
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Abstract

The characterisation of f-vectors of polytopes of arbitrary dimension–or more generally of their flag vectors–is one of the major unsolved problems in discrete geometry. Solutions to this problem in the special cases of 3-dimensional polytopes and simplicial polytopes of arbitrary dimension are Steinitz' theorem and the g-theorem, respectively, but the general problem remains yet unsolved.

For comparatively small dimensions however, f- and flag vectors live in a reasonably low-dimensional space and can therefore be visualised by rather simple methods. We show visualisations of f-vectors of 4- and 5-polytopes, as well as of flag vectors of 4-polytopes. Additionally, we present a recent result and some conjectures related to the topic.

INCLUSION MATRIX AND COHOMOLOGY GROUP

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Abstract

Given a simplicial complex K, namely a set system closed under the taking subset operation, we consider the binary space F_2^K generated by elements of K. The inclusion operator Incl on F_2^K is a linear transformation which sends each elements of K to the sum of all its proper subsets. We prove that Ker (Incl)/Im (Incl) is the direct sum of all reduced cohomology groups of K. We also deduce several other related combinatorial results and discuss its background in phylogenetic combinatorics.

TROPICAL POLYTOPES AND CELLULAR RESOLUTIONS

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Abstract

Tropical convex hulls of finite sets of points are analogues of usual convex hulls, in the geometry over the tropical semiring ($\mathbb{R}$, min,+). They are usual polyhedral complexes and have natural structures of cellular free resolutions, i.e., their combinatorial data give rise to minimal free resolutions for some monomial ideals. On the other hand, they are also images of usual polytopes in an affine space over the Puiseux series field, under the degree map. This gives rise to a family of cellular resolutions of arbitrary monomial ideals, which includes the hull resolution. We also propose a new definition of faces of tropical polytopes, which leads to several open conjectures about their structure.

INTEGER REALIZATIONS OF STACKED POLYTOPES

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Abstract

By a fundamental theorem of Steinitz, the edge graphs of 3-polytopes are exactly the 3-connected planar graphs. It is also known that every 3-polytope has a realization with integral vertex coordinates. The following question is wide open: How big does a box have to be such that every 3-polytope with n vertices has an integral realization in this box? The currently best general upper bound is 2^{12n^2}, while the best lower bound is $n^{3/2}$.

We consider the class of stacked polytopes, and try to improve on the general bounds for this class. The corresponding graphs - stacked triangulations - can be derived from a K_3 embedded in the plane by stacking vertices of degree three into bounded triangles. We associate a ternary tree with a stacked triangulation T, its leafs are in bijection with the bounded faces of T. We show that stacked triangulations corresponding to a complete tree of height h have an integral realization in an $2n \times 2n \times 8n^3$ box. Furthermore, we show that the stacked triangulations associated with a caterpillar can be realized in an $n \times n \times 4n^4$ box.