

# MATHEMATICS

## Higher 2

### (Syllabus 9740)

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## AIMS

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The syllabus prepares students adequately for university courses including mathematics, physics and engineering, where more mathematics content is required. The syllabus aims to develop mathematical thinking and problem solving skills in students. Topics covered include *Functions and Graphs*, *Sequences and Series*, *Vectors*, *Complex Numbers*, *Calculus*, *Permutations*, *Combinations and Probability*, *Binomial*, *Poisson and Normal Distributions*, *Sampling and Hypothesis Testing*, and *Correlation and Regression*. Students will learn to analyse, formulate and solve different types of problems. They will also learn to work with data and perform statistical analyses.

The general aims of the syllabus are to enable students to:

- acquire the necessary mathematical concepts and skills for everyday life, and for continuous learning in mathematics and related disciplines
- develop the necessary process skills for the acquisition and application of mathematical concepts and skills
- develop the mathematical thinking and problem solving skills and apply these skills to formulate and solve problems
- recognise and use connections among mathematical ideas, and between mathematics and other disciplines
- develop positive attitudes towards mathematics
- make effective use of a variety of mathematical tools (including information and communication technology tools) in the learning and application of mathematics
- produce imaginative and creative work arising from mathematical ideas
- develop the abilities to reason logically, to communicate mathematically, and to learn cooperatively and independently

## ASSESSMENT OBJECTIVES (AO)

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There are three levels of assessment objectives for the examination.

The assessment will test candidates' abilities to:

- AO1** understand and apply mathematical concepts and skills in a variety of contexts, including the manipulation of mathematical expressions and use of graphing calculators
- AO2** reason and communicate mathematically through writing mathematical explanation, arguments and proofs, and inferences
- AO3** solve unfamiliar problems; translate common realistic contexts into mathematics; interpret and evaluate mathematical results, and use the results to make predictions, or comment on the context

## USE OF GRAPHING CALCULATOR (GC)

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The use of GC without computer algebra system will be expected. The examination papers will be set with the assumption that candidates will have access to GC. As a general rule, unsupported answers obtained from GC are allowed unless the question states otherwise. Where unsupported answers from GC are not allowed, candidates are required to present the mathematical steps using mathematical notations and not calculator commands. For questions where graphs are used to find a solution, candidates should sketch these graphs as part of their answers. Incorrect answers without working will receive no marks. However, if there is written evidence of using GC correctly, method marks may be awarded.

Students should be aware that there are limitations inherent in GC. For example, answers obtained by tracing along a graph to find roots of an equation may not produce the required accuracy.

## LIST OF FORMULAE

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Candidates will be provided in the examination with a list of formulae.

## INTEGRATION AND APPLICATION

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Notwithstanding the presentation of the topics in the syllabus document, it is envisaged that some examination questions may integrate ideas from more than one topic, and that topics may be tested in the contexts of problem solving and application of mathematics.

## SCHEME OF EXAMINATION PAPERS

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For the examination in H2 Mathematics, there will be two 3-hour papers, each carrying 50% of the total mark, and each marked out of 100, as follows:

### **PAPER 1 (3 hours)**

A paper consisting of about 10 to 12 questions of different lengths and marks based on the Pure Mathematics section of the syllabus.

Candidates will be expected to answer **all** questions.

### **PAPER 2 (3 hours)**

A paper consisting of 2 sections, Sections A and B.

**Section A** (Pure Mathematics – 40 marks) will consist of about 3–4 questions of different lengths and marks based on the Pure Mathematics section of the syllabus.

**Section B** (Statistics – 60 marks) will consist of about 6–8 questions of different lengths and marks based on the Statistics section of the syllabus.

Candidates will be expected to answer **all** questions.

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## CONTENT OUTLINE

Knowledge of the content of the O Level Mathematics syllabus and of some of the content of the O Level Additional Mathematics syllabus are assumed in the syllabus below and will not be tested directly, but it may be required indirectly in response to questions on other topics. The assumed knowledge for O Level Additional Mathematics is appended after this section.

	Topic/Sub-topics	Content
<b>PURE MATHEMATICS</b>		
<b>1</b>	<b>Functions and graphs</b>	
1.1	Functions, inverse functions and composite functions	<p>Include:</p> <ul style="list-style-type: none"> <li>• concepts of function, domain and range</li> <li>• use of notations such as <math>f(x) = x^2 + 5</math>, <math>f: x \mapsto x^2 + 5</math>, <math>f^{-1}(x)</math>, <math>fg(x)</math> and <math>f^2(x)</math></li> <li>• finding inverse functions and composite functions</li> <li>• conditions for the existence of inverse functions and composite functions</li> <li>• domain restriction to obtain an inverse function</li> <li>• relationship between a function and its inverse as reflection in the line <math>y = x</math></li> </ul> <p>Exclude the use of the relation <math>(fg)^{-1} = g^{-1}f^{-1}</math></p>
1.2	Graphing techniques	<p>Include:</p> <ul style="list-style-type: none"> <li>• use of a graphing calculator to graph a given function</li> <li>• relating the following equations with their graphs           <math display="block">\frac{x^2}{a^2} \pm \frac{y^2}{b^2} = 1</math> <math display="block">y = \frac{ax + b}{cx + d}</math> <math display="block">y = \frac{ax^2 + bx + c}{dx + e}</math> </li> <li>• characteristics of graphs such as symmetry, intersections with the axes, turning points and asymptotes</li> <li>• determining the equations of asymptotes, axes of symmetry, and restrictions on the possible values of <math>x</math> and/or <math>y</math></li> <li>• effect of transformations on the graph of <math>y = f(x)</math> as represented by <math>y = af(x)</math>, <math>y = f(x) + a</math>, <math>y = f(x + a)</math> and <math>y = f(ax)</math>, and combinations of these transformations</li> <li>• relating the graphs of <math>y =  f(x) </math>, <math>y = f( x )</math>, <math>y = \frac{1}{f(x)}</math> and <math>y^2 = f(x)</math> to the graph of <math>y = f(x)</math></li> <li>• simple parametric equations and their graphs</li> </ul>

	Topic/Sub-topics	Content
1.3	Equations and inequalities	Include: <ul style="list-style-type: none"> <li>solving inequalities of the form <math>\frac{f(x)}{g(x)} &gt; 0</math> where <math>f(x)</math> and <math>g(x)</math> are quadratic expressions that are either factorisable or always positive</li> <li>solving inequalities by graphical methods</li> <li>formulating an equation or a system of linear equations from a problem situation</li> <li>finding the numerical solution of equations (including system of linear equations) using a graphing calculator</li> </ul>
<b>2</b>	<b>Sequences and series</b>	
2.1	Summation of series	Include: <ul style="list-style-type: none"> <li>concepts of sequence and series</li> <li>relationship between <math>u_n</math> (the <math>n</math>th term) and <math>S_n</math> (the sum to <math>n</math> terms)</li> <li>sequence given by a formula for the <math>n</math>th term</li> <li>sequence generated by a simple recurrence relation of the form <math>x_{n+1} = f(x_n)</math></li> <li>use of <math>\Sigma</math> notation</li> <li>summation of series by the method of differences</li> <li>convergence of a series and the sum to infinity</li> <li>binomial expansion of <math>(1+x)^n</math> for any rational <math>n</math></li> <li>condition for convergence of a binomial series</li> <li>proof by the method of mathematical induction</li> </ul>
2.2	Arithmetic and geometric series	Include: <ul style="list-style-type: none"> <li>formula for the <math>n</math>th term and the sum of a finite arithmetic series</li> <li>formula for the <math>n</math>th term and the sum of a finite geometric series</li> <li>condition for convergence of an infinite geometric series</li> <li>formula for the sum to infinity of a convergent geometric series</li> <li>solving practical problems involving arithmetic and geometric series</li> </ul>
<b>3</b>	<b>Vectors</b>	
3.1	Vectors in two and three dimensions	Include: <ul style="list-style-type: none"> <li>addition and subtraction of vectors, multiplication of a vector by a scalar, and their geometrical interpretations</li> <li>use of notations such as <math>\begin{pmatrix} x \\ y \end{pmatrix}</math>, <math>\begin{pmatrix} x \\ y \\ z \end{pmatrix}</math>, <math>x\mathbf{i} + y\mathbf{j}</math>, <math>x\mathbf{i} + y\mathbf{j} + z\mathbf{k}</math>, <math>\overrightarrow{AB}</math>, <math>\mathbf{a}</math></li> <li>position vectors and displacement vectors</li> <li>magnitude of a vector</li> <li>unit vectors</li> <li>distance between two points</li> <li>angle between a vector and the <math>x</math>-, <math>y</math>- or <math>z</math>-axis</li> <li>use of the ratio theorem in geometrical applications</li> </ul>

	Topic/Sub-topics	Content
3.2	The scalar and vector products of vectors	<p>Include:</p> <ul style="list-style-type: none"> <li>• concepts of scalar product and vector product of vectors</li> <li>• calculation of the magnitude of a vector and the angle between two directions</li> <li>• calculation of the area of triangle or parallelogram</li> <li>• geometrical meanings of <math> \mathbf{a} \cdot \mathbf{b} </math> and <math> \mathbf{a} \times \mathbf{b} </math>, where <math>\mathbf{b}</math> is a unit vector</li> </ul> <p>Exclude triple products <math>\mathbf{a} \cdot \mathbf{b} \times \mathbf{c}</math> and <math>\mathbf{a} \times \mathbf{b} \times \mathbf{c}</math></p>
3.3	Three-dimensional geometry	<p>Include:</p> <ul style="list-style-type: none"> <li>• vector and cartesian equations of lines and planes</li> <li>• finding the distance from a point to a line or to a plane</li> <li>• finding the angle between two lines, between a line and a plane, or between two planes</li> <li>• relationships between <ul style="list-style-type: none"> <li>– two lines (coplanar or skew)</li> <li>– a line and a plane</li> <li>– two planes</li> <li>– three planes</li> </ul> </li> <li>• finding the intersections of lines and planes</li> </ul> <p>Exclude:</p> <ul style="list-style-type: none"> <li>• finding the shortest distance between two skew lines</li> <li>• finding an equation for the common perpendicular to two skew lines</li> </ul>
<b>4</b>	<b>Complex numbers</b>	
4.1	Complex numbers expressed in cartesian form	<p>Include:</p> <ul style="list-style-type: none"> <li>• extension of the number system from real numbers to complex numbers</li> <li>• complex roots of quadratic equations</li> <li>• four operations of complex numbers expressed in the form <math>(x + iy)</math></li> <li>• equating real parts and imaginary parts</li> <li>• conjugate roots of a polynomial equation with real coefficients</li> </ul>

	Topic/Sub-topics	Content
4.2	Complex numbers expressed in polar form	<p>Include:</p> <ul style="list-style-type: none"> <li>• complex numbers expressed in the form <math>r(\cos\theta + i\sin\theta)</math> or <math>re^{i\theta}</math>, where <math>r &gt; 0</math> and <math>-\pi &lt; \theta \leq \pi</math></li> <li>• calculation of modulus (<math>r</math>) and argument (<math>\theta</math>) of a complex number</li> <li>• multiplication and division of two complex numbers expressed in polar form</li> <li>• representation of complex numbers in the Argand diagram</li> <li>• geometrical effects of conjugating a complex number and of adding, subtracting, multiplying, dividing two complex numbers</li> <li>• loci such as <math> z - c  \leq r</math>, <math> z - a  =  z - b </math> and <math>\arg(z - a) = \alpha</math></li> <li>• use of de Moivre's theorem to find the powers and <math>n</math>th roots of a complex number</li> </ul> <p>Exclude:</p> <ul style="list-style-type: none"> <li>• loci such as <math> z - a  = k z - b </math>, where <math>k \neq 1</math> and <math>\arg(z - a) - \arg(z - b) = \alpha</math></li> <li>• properties and geometrical representation of the <math>n</math>th roots of unity</li> <li>• use of de Moivre's theorem to derive trigonometric identities</li> </ul>
<b>5</b>	<b>Calculus</b>	
5.1	Differentiation	<p>Include:</p> <ul style="list-style-type: none"> <li>• graphical interpretation of <ul style="list-style-type: none"> <li>– <math>f'(x) &gt; 0</math>, <math>f'(x) = 0</math> and <math>f'(x) &lt; 0</math></li> <li>– <math>f''(x) &gt; 0</math> and <math>f''(x) &lt; 0</math></li> </ul> </li> <li>• relating the graph of <math>y = f'(x)</math> to the graph of <math>y = f(x)</math></li> <li>• differentiation of simple functions defined implicitly or parametrically</li> <li>• finding the numerical value of a derivative at a given point using a graphing calculator</li> <li>• finding equations of tangents and normals to curves</li> <li>• solving practical problems involving differentiation</li> </ul> <p>Exclude:</p> <ul style="list-style-type: none"> <li>• finding non-stationary points of inflexion</li> <li>• problems involving small increments and approximation</li> </ul>

	Topic/Sub-topics	Content
5.2	Maclaurin's series	<p>Include:</p> <ul style="list-style-type: none"> <li>derivation of the first few terms of the series expansion of <math>(1+x)^n</math>, <math>e^x</math>, <math>\sin x</math>, <math>\ln(1+x)</math>, and other simple functions</li> <li>finding the first few terms of the series expansions of sums and products of functions, e.g. <math>e^x \cos 2x</math>, using standard series</li> <li>summation of infinite series in terms of standard series</li> <li><math>\sin x \approx x</math>, <math>\cos x \approx 1 - \frac{1}{2}x^2</math>, <math>\tan x \approx x</math></li> <li>concepts of 'convergence' and 'approximation'</li> </ul> <p>Exclude derivation of the general term of the series</p>
5.3	Integration techniques	<p>Include:</p> <ul style="list-style-type: none"> <li>integration of <math>\frac{f'(x)}{f(x)}</math></li> <li><math>\sin^2 x</math>, <math>\cos^2 x</math>, <math>\tan^2 x</math></li> <li><math>\frac{1}{a^2 + x^2}</math>, <math>\frac{1}{\sqrt{a^2 - x^2}}</math>, <math>\frac{1}{a^2 - x^2}</math> and <math>\frac{1}{x^2 - a^2}</math></li> <li>integration by a given substitution</li> <li>integration by parts</li> </ul> <p>Exclude reduction formulae</p>
5.4	Definite integrals	<p>Include:</p> <ul style="list-style-type: none"> <li>concept of definite integral as a limit of sum</li> <li>definite integral as the area under a curve</li> <li>evaluation of definite integrals</li> <li>finding the area of a region bounded by a curve and lines parallel to the coordinate axes, between a curve and a line, or between two curves</li> <li>area below the x-axis</li> <li>finding the area under a curve defined parametrically</li> <li>finding the volume of revolution about the x- or y-axis</li> <li>finding the numerical value of a definite integral using a graphing calculator</li> </ul> <p>Exclude approximation of area under a curve using the trapezium rule</p>



	Topic/Sub-topics	Content
5.5	Differential equations	Include: <ul style="list-style-type: none"> <li>solving differential equations of the forms               <math display="block">\frac{dy}{dx} = f(x)</math> <math display="block">\frac{dy}{dx} = f(y)</math> <math display="block">\frac{d^2y}{dx^2} = f(x)</math> </li> <li>formulating a differential equation from a problem situation</li> <li>use of a family of solution curves to represent the general solution of a differential equation</li> <li>use of an initial condition to find a particular solution</li> <li>interpretation of a solution in terms of the problem situation</li> </ul>
<b>STATISTICS</b>		
<b>6</b>	<b>Permutations, combinations and probability</b>	
6.1	Permutations and combinations	Include: <ul style="list-style-type: none"> <li>addition and multiplication principles for counting</li> <li>concepts of permutation (<math>n!</math> or <math>{}^n P_r</math>) and combination (<math>{}^n C_r</math>)</li> <li>arrangements of objects in a line or in a circle</li> <li>cases involving repetition and restriction</li> </ul>
6.2	Probability	Include: <ul style="list-style-type: none"> <li>addition and multiplication of probabilities</li> <li>mutually exclusive events and independent events</li> <li>use of tables of outcomes, Venn diagrams, and tree diagrams to calculate probabilities</li> <li>calculation of conditional probabilities in simple cases</li> <li>use of:               <math display="block">P(A') = 1 - P(A)</math> <math display="block">P(A \cup B) = P(A) + P(B) - P(A \cap B)</math> <math display="block">P(A B) = \frac{P(A \cap B)}{P(B)}</math> </li> </ul>
<b>7</b>	<b>Binomial, Poisson and normal distributions</b>	
7.1	Binomial and Poisson distributions	Include: <ul style="list-style-type: none"> <li>concepts of binomial distribution <math>B(n, p)</math> and Poisson distribution <math>Po(\mu)</math>; use of <math>B(n, p)</math> and <math>Po(\mu)</math> as probability models</li> <li>use of mean and variance of binomial and Poisson distributions (without proof)</li> <li>solving problems involving binomial and Poisson variables</li> <li>additive property of the Poisson distribution</li> <li>Poisson approximation to binomial</li> </ul> Exclude calculation of mean and variance for other probability distributions

	Topic/Sub-topics	Content
7.2	Normal distribution	<p>Include:</p> <ul style="list-style-type: none"> <li>• concept of a normal distribution and its mean and variance; use of <math>N(\mu, \sigma^2)</math> as a probability model</li> <li>• standard normal distribution</li> <li>• finding the value of <math>P(X &lt; x_1)</math> given the values of <math>x_1, \mu, \sigma</math></li> <li>• use of the symmetry of the normal distribution</li> <li>• finding a relationship between <math>x_1, \mu, \sigma</math> given the value of <math>P(X &lt; x_1)</math></li> <li>• solving problems involving normal variables</li> <li>• solving problems involving the use of <math>E(aX + b)</math> and <math>\text{Var}(aX + b)</math></li> <li>• solving problems involving the use of <math>E(aX + bY)</math> and <math>\text{Var}(aX + bY)</math>, where <math>X</math> and <math>Y</math> are independent</li> <li>• normal approximation to binomial</li> <li>• normal approximation to Poisson</li> </ul> <p>Exclude:</p> <ul style="list-style-type: none"> <li>• finding probability density functions and distribution functions</li> <li>• calculation of <math>E(X)</math> and <math>\text{Var}(X)</math> from other probability density functions</li> </ul>
<b>8</b>	<b>Sampling and hypothesis testing</b>	
8.1	Sampling	<p>Include:</p> <ul style="list-style-type: none"> <li>• concepts of population and sample</li> <li>• random, stratified, systematic and quota samples</li> <li>• advantages and disadvantages of the various sampling methods</li> <li>• distribution of sample means from a normal population</li> <li>• use of the Central Limit Theorem to treat sample means as having normal distribution when the sample size is sufficiently large</li> <li>• calculation of unbiased estimates of the population mean and variance from a sample</li> <li>• solving problems involving the sampling distribution</li> </ul>
8.2	Hypothesis testing	<p>Include:</p> <ul style="list-style-type: none"> <li>• concepts of null and alternative hypotheses, test statistic, level of significance and <math>p</math>-value</li> <li>• tests for a population mean based on: <ul style="list-style-type: none"> <li>– a sample from a normal population of known variance</li> <li>– a sample from a normal population of unknown variance</li> <li>– a large sample from any population</li> </ul> </li> <li>• 1-tail and 2-tail tests</li> <li>• use of <math>t</math>-test</li> </ul> <p>Exclude testing the difference between two population means</p>

	Topic/Sub-topics	Content
<b>9</b>	<b>Correlation and Regression</b>	
9.1	Correlation coefficient and linear regression	<p>Include:</p> <ul style="list-style-type: none"> <li>• concepts of scatter diagram, correlation coefficient and linear regression</li> <li>• calculation and interpretation of the product moment correlation coefficient and of the equation of the least squares regression line</li> <li>• concepts of interpolation and extrapolation</li> <li>• use of a square, reciprocal or logarithmic transformation to achieve linearity</li> </ul> <p>Exclude:</p> <ul style="list-style-type: none"> <li>• derivation of formulae</li> <li>• hypothesis tests</li> </ul>

## ASSUMED KNOWLEDGE

Content from O Level Additional Mathematics	
A	ALGEBRA
1	Equations and inequalities <ul style="list-style-type: none"> <li>• conditions for a quadratic equation to have:               <ul style="list-style-type: none"> <li>– two real roots</li> <li>– two equal roots</li> <li>– no real roots</li> </ul> </li> <li>• conditions for <math>ax^2 + bx + c</math> to be always positive (or always negative)</li> </ul>
2	Polynomials <ul style="list-style-type: none"> <li>• multiplication and division of polynomials</li> <li>• use of remainder and factor theorems</li> </ul>
3	Indices and surds <ul style="list-style-type: none"> <li>• four operations on surds</li> <li>• rationalising the denominator</li> </ul>
4	Simultaneous equations in two unknowns <ul style="list-style-type: none"> <li>• solving simultaneous equations with at least one linear equation, by substitution</li> <li>• express a pair of linear equations in matrix form and solving the equations by inverse matrix method</li> </ul>
5	Exponential and logarithmic functions <ul style="list-style-type: none"> <li>• functions <math>a^x</math>, <math>e^x</math>, <math>\log_a x</math>, <math>\ln x</math> and their graphs</li> <li>• laws of logarithms</li> <li>• equivalence of <math>y = a^x</math> and <math>x = \log_a y</math></li> <li>• change of base of logarithms</li> <li>• function <math> x </math> and graph of <math> f(x) </math>, where <math>f(x)</math> is linear, quadratic or trigonometric</li> <li>• solving simple equations involving exponential and logarithmic functions</li> </ul>
6	Partial fractions <ul style="list-style-type: none"> <li>• Include cases where the denominator is of the form               <ul style="list-style-type: none"> <li>– <math>(ax + b)(cx + d)</math></li> <li>– <math>(ax + b)(cx + d)^2</math></li> <li>– <math>(ax + b)(x^2 + c^2)</math></li> </ul> </li> </ul>

<b>Content from O Level Additional Mathematics</b>	
<b>B</b>	<b>GEOMETRY AND TRIGONOMETRY</b>
7	<p>Coordinate geometry in two dimensions</p> <ul style="list-style-type: none"> <li>graphs of equations <ul style="list-style-type: none"> <li><math>y = ax^n</math>, where <math>n</math> is a simple rational number</li> <li><math>y^2 = kx</math></li> </ul> </li> <li>coordinate geometry of the circle with the equation in the form <math>(x - a)^2 + (y - b)^2 = r^2</math> or <math>x^2 + y^2 + 2gx + 2fy + c = 0</math></li> </ul>
8	<p>Trigonometry</p> <ul style="list-style-type: none"> <li>six trigonometric functions, and principal values of the inverses of sine, cosine and tangent</li> <li>trigonometric equations and identities (see List of Formulae)</li> <li>expression of <math>a \cos \theta + b \sin \theta</math> in the forms <math>R \sin(\theta \pm \alpha)</math> and <math>R \cos(\theta \pm \alpha)</math></li> </ul>
<b>C</b>	<b>CALCULUS</b>
9	<p>Differentiation and integration</p> <ul style="list-style-type: none"> <li>derivative of <math>f(x)</math> as the gradient of the tangent to the graph of <math>y = f(x)</math> at a point</li> <li>derivative as rate of change</li> <li>derivatives of <math>x^n</math> for any rational <math>n</math>, <math>\sin x</math>, <math>\cos x</math>, <math>\tan x</math>, <math>e^x</math> and <math>\ln x</math>, together with constant multiples, sums and differences</li> <li>derivatives of composite functions</li> <li>derivatives of products and quotients of functions</li> <li>increasing and decreasing functions</li> <li>stationary points (maximum and minimum turning points and points of inflexion)</li> <li>use of second derivative test to discriminate between maxima and minima</li> <li>connected rates of change</li> <li>maxima and minima problems</li> <li>integration as the reverse of differentiation</li> <li>integration of <math>x^n</math> for any rational <math>n</math>, <math>e^x</math>, <math>\sin x</math>, <math>\cos x</math>, <math>\sec^2 x</math> and their constant multiples, sums and differences</li> <li>integration of <math>(ax + b)^n</math> for any rational <math>n</math>, <math>\sin(ax + b)</math>, <math>\cos(ax + b)</math> and <math>e^{ax+b}</math></li> </ul>

# MATHEMATICAL NOTATION

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The list which follows summarises the notation used in Cambridge's Mathematics examinations. Although primarily directed towards A Level, the list also applies, where relevant, to examinations at all other levels.

## 1. Set Notation

$\in$	is an element of
$\notin$	is not an element of
$\{x_1, x_2, \dots\}$	the set with elements $x_1, x_2, \dots$
$\{x: \dots\}$	the set of all $x$ such that
$n(A)$	the number of elements in set $A$
$\emptyset$	the empty set
$\mathcal{U}$	universal set
$A'$	the complement of the set $A$
$\mathbb{Z}$	the set of integers, $\{0, \pm 1, \pm 2, \pm 3, \dots\}$
$\mathbb{Z}^+$	the set of positive integers, $\{1, 2, 3, \dots\}$
$\mathbb{Q}$	the set of rational numbers
$\mathbb{Q}^+$	the set of positive rational numbers, $\{x \in \mathbb{Q}: x > 0\}$
$\mathbb{Q}_0^+$	the set of positive rational numbers and zero, $\{x \in \mathbb{Q}: x \geq 0\}$
$\mathbb{R}$	the set of real numbers
$\mathbb{R}^+$	the set of positive real numbers, $\{x \in \mathbb{R}: x > 0\}$
$\mathbb{R}_0^+$	the set of positive real numbers and zero, $\{x \in \mathbb{R}: x \geq 0\}$
$\mathbb{R}^n$	the real $n$ tuples
$\mathbb{C}$	the set of complex numbers
$\subseteq$	is a subset of
$\subset$	is a proper subset of
$\not\subseteq$	is not a subset of
$\not\subset$	is not a proper subset of
$\cup$	union
$\cap$	intersection
$[a, b]$	the closed interval $\{x \in \mathbb{R}: a \leq x \leq b\}$
$[a, b)$	the interval $\{x \in \mathbb{R}: a \leq x < b\}$
$(a, b]$	the interval $\{x \in \mathbb{R}: a < x \leq b\}$
$(a, b)$	the open interval $\{x \in \mathbb{R}: a < x < b\}$

## 2. Miscellaneous Symbols

$=$	is equal to
$\neq$	is not equal to
$\equiv$	is identical to or is congruent to
$\approx$	is approximately equal to
$\propto$	is proportional to
$<$	is less than
$\leq; \nlessgtr$	is less than or equal to; is not greater than
$>$	is greater than
$\geq; \ngtr$	is greater than or equal to; is not less than
$\infty$	infinity

## 3. Operations

$a + b$	$a$ plus $b$
$a - b$	$a$ minus $b$
$a \times b, ab, a.b$	$a$ multiplied by $b$
$a \div b, \frac{a}{b}, a/b$	$a$ divided by $b$
$a : b$	the ratio of $a$ to $b$
$\sum_{i=1}^n a_i$	$a_1 + a_2 + \dots + a_n$
$\sqrt{a}$	the positive square root of the real number $a$
$ a $	the modulus of the real number $a$
$n!$	$n$ factorial for $n \in \mathbb{Z}^+ \cup \{0\}$ , ( $0! = 1$ )
$\binom{n}{r}$	the binomial coefficient $\frac{n!}{r!(n-r)!}$ , for $n, r \in \mathbb{Z}^+ \cup \{0\}$ , $0 \leq r \leq n$ $\frac{n(n-1)\dots(n-r+1)}{r!}$ , for $n \in \mathbb{Q}$ , $r \in \mathbb{Z}^+ \cup \{0\}$

## 4. Functions

$f$	function $f$
$f(x)$	the value of the function $f$ at $x$
$f: A \rightarrow B$	$f$ is a function under which each element of set $A$ has an image in set $B$
$f: x \mapsto y$	the function $f$ maps the element $x$ to the element $y$
$f^{-1}$	the inverse of the function $f$
$g \circ f, gf$	the composite function of $f$ and $g$ which is defined by $(g \circ f)(x)$ or $gf(x) = g(f(x))$
$\lim_{x \rightarrow a} f(x)$	the limit of $f(x)$ as $x$ tends to $a$
$\Delta x; \delta x$	an increment of $x$
$\frac{dy}{dx}$	the derivative of $y$ with respect to $x$
$\frac{d^n y}{dx^n}$	the $n$ th derivative of $y$ with respect to $x$
$f'(x), f''(x), \dots, f^{(n)}(x)$	the first, second, ... $n$ th derivatives of $f(x)$ with respect to $x$
$\int y dx$	indefinite integral of $y$ with respect to $x$
$\int_a^b y dx$	the definite integral of $y$ with respect to $x$ for values of $x$ between $a$ and $b$
$\dot{x}, \ddot{x}, \dots$	the first, second, ... derivatives of $x$ with respect to time

## 5. Exponential and Logarithmic Functions

$e$	base of natural logarithms
$e^x, \exp x$	exponential function of $x$
$\log_a x$	logarithm to the base $a$ of $x$
$\ln x$	natural logarithm of $x$
$\lg x$	logarithm of $x$ to base 10

## 6. Circular Functions and Relations

$\sin, \cos, \tan,$ $\operatorname{cosec}, \sec, \cot$	} the circular functions
$\sin^{-1}, \cos^{-1}, \tan^{-1}$ $\operatorname{cosec}^{-1}, \sec^{-1}, \cot^{-1}$	} the inverse circular functions



## 7. Complex Numbers

$i$	square root of $-1$
$z$	a complex number, $z = x + iy$ $= r(\cos \theta + i \sin \theta), r \in \mathbb{R}_0^+$ $= re^{i\theta}, r \in \mathbb{R}_0^+$
$\operatorname{Re} z$	the real part of $z$ , $\operatorname{Re}(x + iy) = x$
$\operatorname{Im} z$	the imaginary part of $z$ , $\operatorname{Im}(x + iy) = y$
$ z $	the modulus of $z$ , $ x + iy  = \sqrt{x^2 + y^2},  r(\cos \theta + i \sin \theta)  = r$
$\arg z$	the argument of $z$ , $\arg(r(\cos \theta + i \sin \theta)) = \theta, -\pi < \theta \leq \pi$
$z^*$	the complex conjugate of $z$ , $(x + iy)^* = x - iy$

## 8. Matrices

$\mathbf{M}$	a matrix $\mathbf{M}$
$\mathbf{M}^{-1}$	the inverse of the square matrix $\mathbf{M}$
$\mathbf{M}^T$	the transpose of the matrix $\mathbf{M}$
$\det \mathbf{M}$	the determinant of the square matrix $\mathbf{M}$

## 9. Vectors

$\mathbf{a}$	the vector $\mathbf{a}$
$\overrightarrow{AB}$	the vector represented in magnitude and direction by the directed line segment $AB$
$\hat{\mathbf{a}}$	a unit vector in the direction of the vector $\mathbf{a}$
$\mathbf{i}, \mathbf{j}, \mathbf{k}$	unit vectors in the directions of the cartesian coordinate axes
$ \mathbf{a} $	the magnitude of $\mathbf{a}$
$ \overrightarrow{AB} $	the magnitude of $\overrightarrow{AB}$
$\mathbf{a} \cdot \mathbf{b}$	the scalar product of $\mathbf{a}$ and $\mathbf{b}$
$\mathbf{a} \times \mathbf{b}$	the vector product of $\mathbf{a}$ and $\mathbf{b}$

## 10. Probability and Statistics

$A, B, C, \text{ etc.}$	events
$A \cup B$	union of events $A$ and $B$
$A \cap B$	intersection of the events $A$ and $B$
$P(A)$	probability of the event $A$
$A'$	complement of the event $A$ , the event 'not $A$ '
$P(A   B)$	probability of the event $A$ given the event $B$
$X, Y, R, \text{ etc.}$	random variables
$x, y, r, \text{ etc.}$	value of the random variables $X, Y, R, \text{ etc.}$
$x_1, x_2, \dots$	observations
$f_1, f_2, \dots$	frequencies with which the observations, $x_1, x_2 \dots$ occur

$p(x)$	the value of the probability function $P(X = x)$ of the discrete random variable $X$
$p_1, p_2 \dots$	probabilities of the values $x_1, x_2, \dots$ of the discrete random variable $X$
$f(x), g(x) \dots$	the value of the probability density function of the continuous random variable $X$
$F(x), G(x) \dots$	the value of the (cumulative) distribution function $P(X \leq x)$ of the random variable $X$
$E(X)$	expectation of the random variable $X$
$E[g(X)]$	expectation of $g(X)$
$\text{Var}(X)$	variance of the random variable $X$
$B(n, p)$	binominal distribution, parameters $n$ and $p$
$\text{Po}(\mu)$	Poisson distribution, mean $\mu$
$N(\mu, \sigma^2)$	normal distribution, mean $\mu$ and variance $\sigma^2$
$\mu$	population mean
$\sigma^2$	population variance
$\sigma$	population standard deviation
$\bar{x}$	sample mean
$s^2$	unbiased estimate of population variance from a sample,
	$s^2 = \frac{1}{n-1} \sum (x - \bar{x})^2$
$\phi$	probability density function of the standardised normal variable with distribution $N(0, 1)$
$\Phi$	corresponding cumulative distribution function
$\rho$	linear product-moment correlation coefficient for a population
$r$	linear product-moment correlation coefficient for a sample