ADDITIONAL MATHEMATICS 4047/02

Paper 2

October/November 2015

2 hours 30 minutes

Answer **all** the questions.

1 The curve $y = \mathrm{f}(x)$ is such that $\mathrm{f}'(x) = 2\mathrm{e}^{x} + \mathrm{e}^{-2x}$.

(i) Explain why the curve $y = \mathrm{f}(x)$ has no stationary points. [2]

(ii) Given that the curve passes through the point (0, 2), find an expression for $\mathrm{f}(x)$. [4]

2 (i) Show that $\frac{\mathrm{d}}{\mathrm{d}x}(\ln(\cos x)) = -\tan x$. [2]

(ii) Differentiate $x \tan x$ with respect to x. [2]

(iii) Using the results from parts (i) and (ii), find $\int x \sec^2 x \, \mathrm{d}x$ and hence show that $\int_0^{\frac{\pi}{4}} x \sec^2 x \, \mathrm{d}x = \frac{\pi}{4} - \frac{1}{2}\ln 2$. [4]

3 The equation of a curve is $y = -x^2 + 4x - 6$. The point P lies on the curve and has an x-coordinate of 1. The tangent to the curve at P meets the x-axis at A and the y-axis at B.

(i) Find the area of triangle AOB, where O is the origin. [6]

The point Q also lies on the curve. The normal to the curve at Q is parallel to the tangent to the curve at P.

(ii) Find the coordinates of Q. [3]

4 (a) (i) Write down, and simplify, the first 4 terms in the expansion of $(1 + x)^9$ in ascending powers of x. [2]

(ii) Replacing x by $z - z^2$, determine the coefficient of z^3 in the expansion of $(1 + z - z^2)^9$. [3]

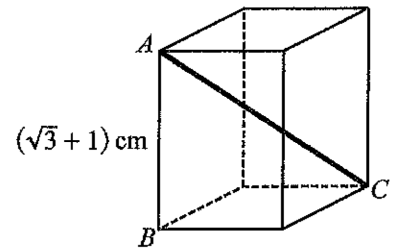
(b) (i) Write down the general term in the binomial expansion of $\left(2x + \frac{1}{3x^3}\right)^{10}$. [1]

(ii) Write down the power of x in this general term. [1]

(iii) Hence, or otherwise, determine the coefficient of x^2 in the binomial expansion of $\left(2x + \frac{1}{3x^3}\right)^{10}$. [2]

5 **Do not use a calculator in this question.**

(i) Express $\frac{11\sqrt{3}}{2\sqrt{3}+1}$ in the form $a + b\sqrt{3}$, where a and b are integers. [2]



The diagram shows a cuboid with a square base. The height AB of the cuboid is $(\sqrt{3} + 1)$ cm.

Given that the length of the diagonal AC is $\frac{11\sqrt{3}}{2\sqrt{3}+1}$ cm,

(ii) find an expression for BC^2 in the form $c + d\sqrt{3}$, where c and d are integers, [3]

(iii) express the volume of the cuboid in the form $\frac{7}{2}(3\sqrt{3} + k)\,\mathrm{cm}^3$, where k is an integer. [4]

6 The equation of a curve is $y = \frac{2x^2}{x-1}$.

(i) Find an expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$ and obtain the coordinates of the stationary points of the curve. [5]

(ii) Find an expression for $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$ and hence determine the nature of these stationary points. [4]

7 The positive x- and y-axes are tangents to a circle C.

(i) What can be deduced about the coordinates of the centre of C? [1]

The line T is a tangent to C at the point (9, 8) on the circle. Given that the centre of C lies below and to the left of (9, 8), find

(ii) the equation of C, [5]

(iii) the equation of T. [3]

8 (i) Find the remainder when $2x^3 - 3x^2 - 5$ is divided by $2x + 1$. [2]

(ii) Factorise completely the cubic polynomial $2x^3 - 3x^2 + 1$. [4]

(iii) Express $\frac{4 - 5x - 8x^2}{2x^3 - 3x^2 + 1}$ as the sum of 3 partial fractions. [4]

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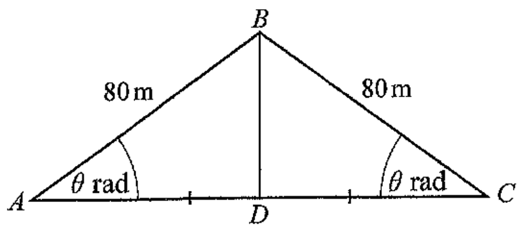

A farmer fences part of his land. He puts fences around the perimeter of the triangular field ABC and also from B to D, where D is the mid-point of AC. Angle BAC = angle $BCA = \theta$ radians and the lengths of AB and BC are 80 m.

(i) Show that L m, the length of fencing needed, can be expressed in the form $p + q\sin\theta + r\cos\theta$, where p, q and r are constants to be found. [3]

(ii) Express L in the form $p + R\sin(\theta + \alpha)$, where $R > 0$ and α is an acute angle. [4]

(iii) Given that the farmer uses exactly 310 m of fencing, find the value of θ. [3]

10 The roots of the quadratic equation $2x^2 - 6x + 5 = 0$ are α and β.

(i) Find the value of $\alpha^2 + \beta^2$. [3]

(ii) Show that the value of $\alpha^3 + \beta^3$ is $\frac{9}{2}$. [2]

(iii) Find a quadratic equation whose roots are $\alpha^2 + \beta$ and $\alpha + \beta^2$. [5]

11 A cuboid of volume V cm^3 has a height of x cm and a rectangular base of area $(px^2 + q)$ cm^2. Corresponding values of x and V are shown in the table below.

x	5	10	15	20
V	175	650	1725	3700

(i) Using suitable variables, draw, on graph paper, a straight line graph and hence estimate the value of each of the constants p and q. [6]

(ii) Using your values of p and q, calculate the value of x for which the cuboid is a cube. [2]

(iii) Explain how another straight line drawn on your diagram can lead to an estimate of the value of x for which the cuboid is a cube. Draw this line and hence verify your value of x found in part **(ii)**. [3]

Answer **all** the questions.

1 Find the value of k for which the coefficient of x^3 in the expansion of $(2 - kx)^5 + (3 + x)^6$ is 860. [5]

2 The acute angles A and B are such that $\tan(A + B) = 8$ and $\tan B = 3$. Without using a calculator, find the exact value of $\sin A$. [5]

3 A particle moves along the curve $y = 2 - \frac{1}{x^2}$ in such a way that the y-coordinate of the particle is increasing at a constant rate of 0.03 units per second. Find the y-coordinate of the particle at the instant that the x-coordinate of the particle is increasing at 0.12 units per second. [5]

4 Express $\frac{(x+2)^2}{x^2(x-2)}$ as the sum of 3 partial fractions. [6]

5 An experiment in Physics to find the focal length, f m, of a lens, requires the student to place an object at a distance, u m, from the lens and to record the distance, v m, at which the image is seen on the other side of the lens. The table below shows some results.

u	0.150	0.200	0.250	0.300
v	0.603	0.299	0.263	0.201

It is known that u, v and f are related by the equation $\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$. It is believed that an error was made in recording one of the values of v.

(i) Plot $\frac{1}{v}$ against $\frac{1}{u}$ and hence determine which value of v, in the table above, is the incorrect recording. [2]

(ii) Draw the straight line graph and use it to estimate a value of v to replace the incorrect recording of v found in part (i). [2]

(iii) Estimate the value of f. [2]

6 (i) Prove that $\frac{1}{(1 + \operatorname{cosec}\theta)(\sec\theta - \tan\theta)} = \tan\theta$. [4]

(ii) Find, in radians, the acute angle for which $\frac{1}{(1 + \operatorname{cosec}\theta)(\sec\theta - \tan\theta)} = 3\cot\theta$. [2]